Error Analysis in a Nutshell

Systematic vs. Statistical (Random) Errors: (Taylor 4.1, 4.6)

Statistical (Random) errors can be reduced by repeating a measurement so the random fluctuations average to zero. Statistical techniques (Std. Deviation, Std. Error, Error Propagation, etc.) apply to statistical errors.

Systematic errors are some difference between our model of the system and the physical system. They are reproducible so they do not average to zero. They can't be improved by repeated measurements and each case must be handled differently.

Standard form: Give the uncertainty to 1 significant figure. Include another digit if the first is a 1. Then give the value in the same format as the uncertainty.

Uncertainty in a single parameter (Taylor 2.1-2.3, 2.8)

Single Measurement:

Estimate the uncertainty as best you can. Note that the uncertainty is usually larger than your ability to read the scale.

Sig Figs! One digit uncertainty. Quote value to same precision as δ.

Multiple measurements:

Standard Deviation: (Taylor 4.2-4.3)

The standard deviation gives the expected uncertainty for a single measurement. It also describes the scatter in values from many repeated measurements. Use STDEV() in Excel or a calculator.

Standard Error: (Taylor 4.4)

The standard error is the uncertainty in the average of repeated measurements. You would expect the average of N measurements to be more accurate than a single measurement, but for a random process, 2 measurements is not twice as good as 1 measurement. Repeating a measurement N times reduces your uncertainty by a factor of $\frac{1}{\sqrt{N}}$.

$$\text{Std. Error} = \frac{1}{\sqrt{N}}\,\text{Std. Dev.}$$

For a slope:

use the LINEST function in Excel

Select a two-by-two grid

=LINEST(y values, x-values, true, true)

(first “true”: also fit intercept, second “true”: calculate errors)

Control-shift-return

Slope	y-intercept
Slope uncertainty	y-intercept uncertainty

Error Propagation: (Taylor 3.5-3.8)

Errors can be propagated through any equation using only three rules.

- For $q = x+y$ or $x-y$, $\delta q = \sqrt{\delta x^2 + \delta y^2}$
- For $q = xy$ or x/y, $\frac{\delta q}{q} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2}$
- For $q = f(x)$, $\delta q = \left|\frac{df(x)}{dx}\right| \delta x$

For more complex equations, these rules can be applied in steps.

For $q= (x+y)*z$, find the uncertainty in the quantity $(x+y)$ first, then use the product rule to combine $(x+y) \pm\sigma_{(x+y)}$ times $z \pm\sigma_z$.

Uncertainty Review Guide

Taylor, "An Introduction to Error Analysis."

These topics will be essential throughout the semester.

Significant Figures:		
Basic Sig Figs:	Sect 2.1-2.3	
Uncertainty more accurate than sig figs:	Sect 2.8	
Propagation of errors:		
Add/Subtract:	Sect. 3.5	Eqn. 3.16
Multiply/Divide:	Sect. 3.6	Eqn. 3.18
Function of 1 Variable:	Sect. 3.7	Eqn. 3.23
"Chain Rule", Dominant Errors:	Sect. 3.8	
Standard Deviations:		
Counting Statistics:	Sect. 3.2	Eqn. 3.2
Standard Deviation:	Sect. 4.2, 4.3	use calculator
Standard Error (Stand. Dev. of the Mean)	Sect. 4.4	Eqn. 4.14
Systematic Errors:	Sect. 4.1, 4.6	
Advanced Topics (Look up when necessary)		
Rejecting Data:	Ch. 6	
Weighted Average:	Ch. 7	